Neural Network Quantum States for Fractional Quantum Hall Physics in the High-Flux Regime of the Bose–Hofstadter Model

F. Tognocchi[1], **D. Piccioni**[2], **N. Regnault**[3], **and F. Becca**[1]

[1] *Dipartimento di Fisica, Università di Trieste, Strada Costiera 11, 34151 Trieste, Italy*
[2]*International School for Advanced Studies (SISSA), Via Bonomea 265, I-34136 Trieste, Italy*
[3]*Center for Computational Quantum Physics, Flatiron Institute, 162 5th Avenue, New York, NY 10010, USA*

In this project, we study the phase diagram of the paradigmatic Hofstadter model [1]. We focus on interacting bosons on a square lattice, with periodic boundary conditions, in the presence of a uniform perpendicular magnetic field.

In the low magnetic flux regime, energy bands are nearly dispersionless, mimicking continuum Landau levels. In this regime, the continuum Laughlin wave function on the torus [2] provides an accurate approximation even for the lattice ground state [3].

As the magnetic flux increases, lattice effects become prominent, causing bands to acquire significant dispersion and rendering continuum-based models unreliable. This high-flux regime remains relatively unexplored and may host interesting physics, as non-flat topological bands have a characteristic energy scale that competes with the interaction scale.

To address both regimes, we employ Neural Network Quantum States (NNQS) [4]. We demonstrate that continuum Laughlin states on the torus admit an exact mapping to a simple neural-network wave function. Despite this exact representability, we find that optimization remains challenging, consistent with other recent studies [5, 6]. We attribute these difficulties to the limited ability of standard neural-network architectures to capture magnetic representations of lattice symmetry transformations. To overcome this issue, we explicitly restore magnetic translational symmetries via quantum-number projection.

We then optimize the neural-network wave function and benchmark its accuracy against exact diagonalization results, finding reliable performance across both low- and high-flux regimes. Furthermore, the use of quantum-number projectors enables access to excited states and thus to the momentum-resolved excitation spectrum. Using these tools, we investigate the possible emergence of phase transitions from Laughlin-type topological states to charge-density-wave phases, as the interaction strength is varied.

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